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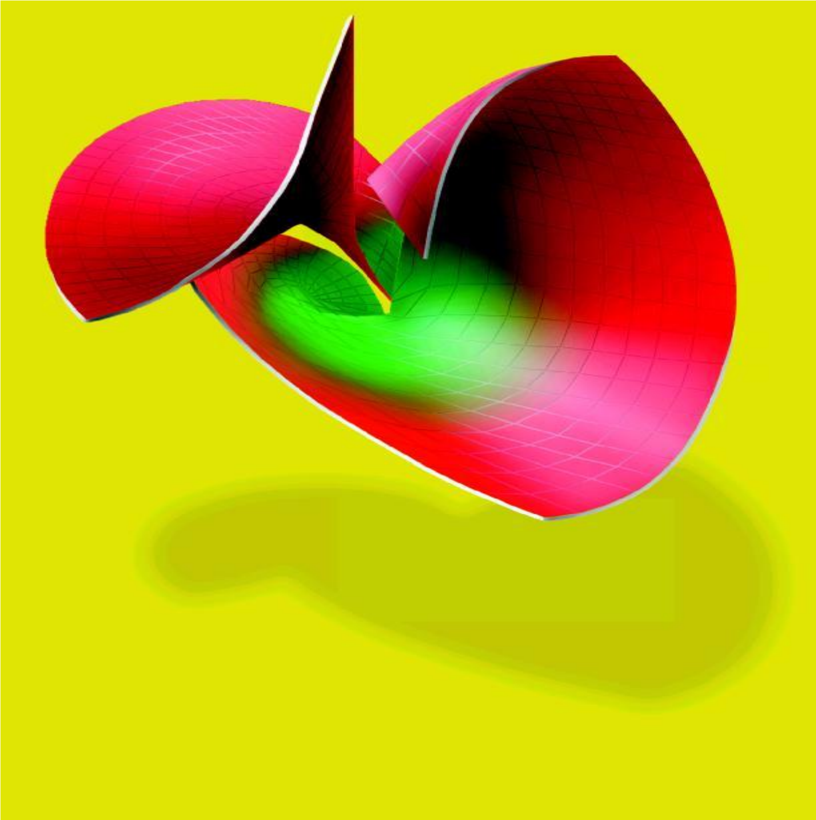
Conservation and Symmetry Laws And Stabilization Programs in Economics

Ben Tamari



Abstract: It is possible to make a good economic forecast. We know that we can make good forecasts in conservative systems, like energy and weather systems, the more closed the system is, the better the forecast. The way to do it is by formulating the **motion equations** and their solutions on the basis of conservative and symmetry laws. These principles are the theoretical basis of the stabilization programs, and the ability to forecast in economics.

Ben Tamari born in Jerusalem in 1942, grow-up in a village, served in the army, and live in Jerusalem since 1964. He was educated in Economic and Philosophy at the Hebrew University, where he presented the thesis for his first book "**Foundations of Economics"** as a PhD dissertation in Economics.

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Dedicated to my mother **Leah,** and my sister **Achsah Ganani**, Z"L.

I express my gratitude to **Batia Harari**, and **Ron Tsur**,
and to the many referees, who remain un-named.

Cover produced by **Mathematica**®

Abstract:

We can make good forecasts in conservative systems (like energy and weather systems).The more closed the system is, the better the forecast. A good economic forecast is also possible, by formulating the *motion equations* and their solutions on the basis of *conservation* and *symmetry* laws. An empirical test verifies the two: the conservation and the symmetry. These properties are the theoretical basis for economic stabilization programs, and the practical basis for economic forecasting (Figure 5, p. 49).

Key Words:

aggregation, allocation, conservation, distribution, growth, indexation, inflation, pricing, printing, rotation, stabilization, symmetry, translation, Bedouin Knot, Gresham Law, Keynes Space, Nominal Anchor, Symmetry Law.

Contents

Introduction

What can be done to bring about the recovery of an autonomous economic system[1] that has gone wrong: decelerating growth, rising unemployment and prices, deficits and debts (domestic and foreign) and declining exchange rates?

An economic system does not fail all at once; the process is protracted, sometimes lasting two or more generations (as in the former Soviet Union). The source of failure is generally a monetary imbalance, relative to the output, which at first produces desirable results, so that the imbalance is maintained, ultimately becoming destructive (somewhat like "pep pills").

By now, sufficient knowledge and experience have been accumulated, on both theoretical and empirical planes, to permit reforms to be introduced into such ailing economic systems. The obstacles to successful reforms are generally political. Namely, strong pressure groups, attempting to protect themselves against what they believe to be harmful effects of the recovery process, hinder implementation of the necessary measures.

One commonly proposed measure for the stabilization of an ailing economic system during planned reforms is what is known today as a "Nominal Anchor", the main component of which is exogenous fixation of the system's rate of exchange [Bruno 1993; Patinkin 1993].

My thesis in this article is that the "Nominal Anchor" may be replaced by another, preferable, dynamic operating rule. This rule, the “Symmetry Law”, applied to an economic system, will steer the system asymptotically to an optimal state. As a result, the system will not have to experience unnecessary shocks after the stage of stabilization and curbing (for example: Israel 1985, Mexico 1988). However, in order for this to happen, the Symmetry Law requires more political resolve from the system than the "Nominal Anchor".

> *The fundamental monetary and fiscal theory that advocates drastic stabilization is probably valid as a rule. To the degree that there are differences between different groups of countries, these are due not to the macroeconomic conceptual framework but rather to problems of budgetary control, quantity of credit or wage policies.* [Bruno 1993, p. 147] (my italics, B.T.).

I would like to present this "*as a rule"* in a more generalized way and characterize the differences between countries as stemming from their different positions in 'Keynes Space' [Q_t, M_t ; t], where Q represents output (G.D.P.), M money (M1), and **t** time.

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[1] An economic system is autonomous if it issues its own currency.

Theory[2]

Conservative Economic System

Money (in real terms) is integrated in present-day macro-economic analysis, in an analytical system, in either of two alternative ways[3]: introduction of money as an argument in the utility function (Tobin and Sidrauski's approach), or introduction of money as an argument in the production function (Patinkin and Levhari's approach) [Blanchard and Fischer 1989, ch. 4; Burmeister and Dobell 1970, ch. 6].

After the dynamic economic system (individual, family, firm or country) has been formalized as a maximization (minimization) problem, one looks for an optimal path, over a certain period of time, that will maximize (minimize) the utility (input) for a given input (utility). These are the primal and dual problems; both may be solved with the tools of Hamiltonian mechanics.

My thesis is that money (in nominal terms) plays a dual role in any economic system: it constitutes both a *measure* and an *input.* Accordingly, any economic monetary system may be described by two fundamental equations, as follows[4].

1. **Money characterized as a measure;**
   This is a concentrated formulation of the quantitative theory: M = PQ, where M denotes money (M1), P the price level (CPI), and Q the output (GDP)[5].

2. **Money characterized as input;**

---

[2] I am quite aware that the brevity of this account and the level of abstraction create an illusion of over-simplification, but here I want only to present the main theme. I hope to present a more comprehensive account at a later time. At present, it is only available in Hebrew [Tamari 1991, 1995].

[3] These two approaches differ conceptually, but are analytically coherent.

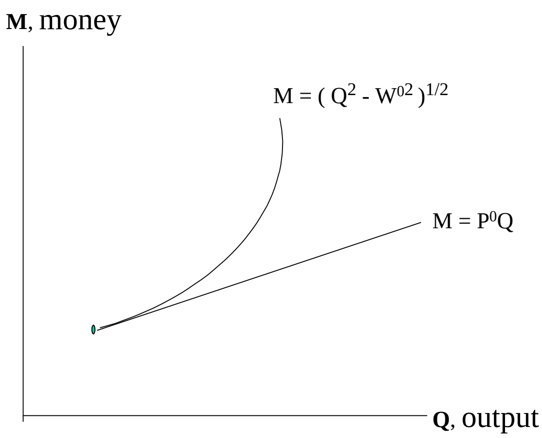
[4] The thesis that money serves both as a *measure* and as *input* (and is therefore not neutral), originates in a feeling that we are still lacking a mathematical way of formalizing 'measures ' (as 'light ' in physics and 'money' in economics) which are revealed as immanent to the empirical system. A necessary (although not always sufficient) condition for obtaining any measure as a systemic measure is that the measure be an integral part of the system itself, so that the system's **nature** will be efficient.

[5] The traditionalists among us may miss the velocity of circulation (V). For our purposes, however, the quantitative equation MV = PQ should be written M = PQ at any given time **t.** Integration over time does not alter the equation, only adds V by definition.

Wealth (W) is a function of the input variables, one of which is money: W = W(K,L,M), where K denotes capital, L labor, and M money. If we denote the production function Q = F(K,L), as is usual in economics literature, we obtain:

$$W = W(Q,M) = (Q^2 - M^2)^{1/2}$$

Figure 1: ***Schematic Description of Keynes Space***



Where W is a kind of 'Potential' on Keynes Space [Q_t, M_t; t][6].

I have coined the Cartesian Product Q x M, which contains the solution paths between two different points in time, as 'Keynes Space' (Figure 1 and 2, p. 17). The actual direction of the path in a given country, at a given time, may be identified by using the Hartman-Grobman Theorem [Guckenheimer and Holmes, 1987, ch. 1].[7]

To that end, we construct two-dimensional linear difference equations. Since these are recursive iterative equations, each **t** is the result of the **t**-1 that precedes it and so on, therefore, one can be satisfied by the current presentation with one lag.

$$\boldsymbol{Q_{t+1}} = \boldsymbol{a\,Q_t} + \boldsymbol{b\,M_t}$$

$$\boldsymbol{M_{t+1}} = \boldsymbol{c\,Q_t} + \boldsymbol{d\,M_t}$$

---

[6] The rationale behind the hyperbolic function is because both direct wisdom and experience testify that when output grows, wealth increases and when money grows (beyond the growth in output), wealth decreases. The square originates in the Pythagorean Theorem, and in the fact that only squared systems survive.

[7] This theorem guarantees that, in the neighborhood of a hyperbolic equilibrium point of a nonlinear function in the phase space, linear approximation will indeed determine the real direction of the motion.

We determine the values of the coefficients **a, b, c,** and **d** (the sensitivities or elasticities or curvatures or flexibilities) for the different countries, by means of logarithmic regressions (in this case in the years 1961 - 1996). We then compute the Jacobian **J,** which is defined as equal to: **ad - bc (= J)**.

After doing a check we find that *on average*, the Jacobian **J** tends to be equal to one (**J** = 1) among the countries, which implies that an economic system in Keynes Space is a *conservative* one because it tends to conserve some quantity (Table 1, p. 28).

A conservative system can be expressed in terms of *complex, exponential,* or *trigonometric* numbers, depending on which of these representations is most convenient for the case at hand.

Complex Version

Wealth (**W**) per economic unit (**j**) at time (**t**) is defined as a complex number with output (**Q**) plotted along the 'real' axis and money (**M**) along the 'imaginary' axis: where **j** denotes an individual, **t** the time, **N** the population and **T** the time horizon.[8]

$$W_{jt} = Q_{jt} + iM_{jt} \qquad 0 \le t \le T, \quad 0 \le j \le N$$

$$W = Q + iM \qquad \textit{aggregation on } t \textit{ and } j$$

Integration of wealth (**W**) will yield the state of the economy, the economic evolution, in the next iteration (year) (**W'**), after some adjustment:

$$W' = \int W dw = 1/2 W^2 + C = 1/2(Q^2 - M^2) + iQM + C$$

Wealth (**W'**) as defined here in Keynes Space, (as a kind of 'Potential'[9]), may be treated with relative ease (numerically and graphically) by using computer software for the solution of (continuous or discrete) dynamical equations.

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[8] A market place represented as a complex space with ad - bc = 1 is a *unimodular group* [Katok, 1992 p. 2, Tamari 1995 p. 49].

[9] This potential ($x^2 - y^2 \pm 2xy$) may be the germ of the D_{-4} Catastrophe: $- V(x, y) = x^2y - 1/3y^3$, (x = Q , y = M) [Gilmore, 1981 p. 503].

Exponential Version

The formal structure that satisfies the conditions of the conservation and the symmetry laws in the exponential version is:

$$p = m - q$$

This is the Pricing Law or Symmetry Law.[10] At any time the next equation must hold.

$$P_0 e^{pt} = \frac{M_0 e^{mt}}{Q_0 e^{qt}}$$

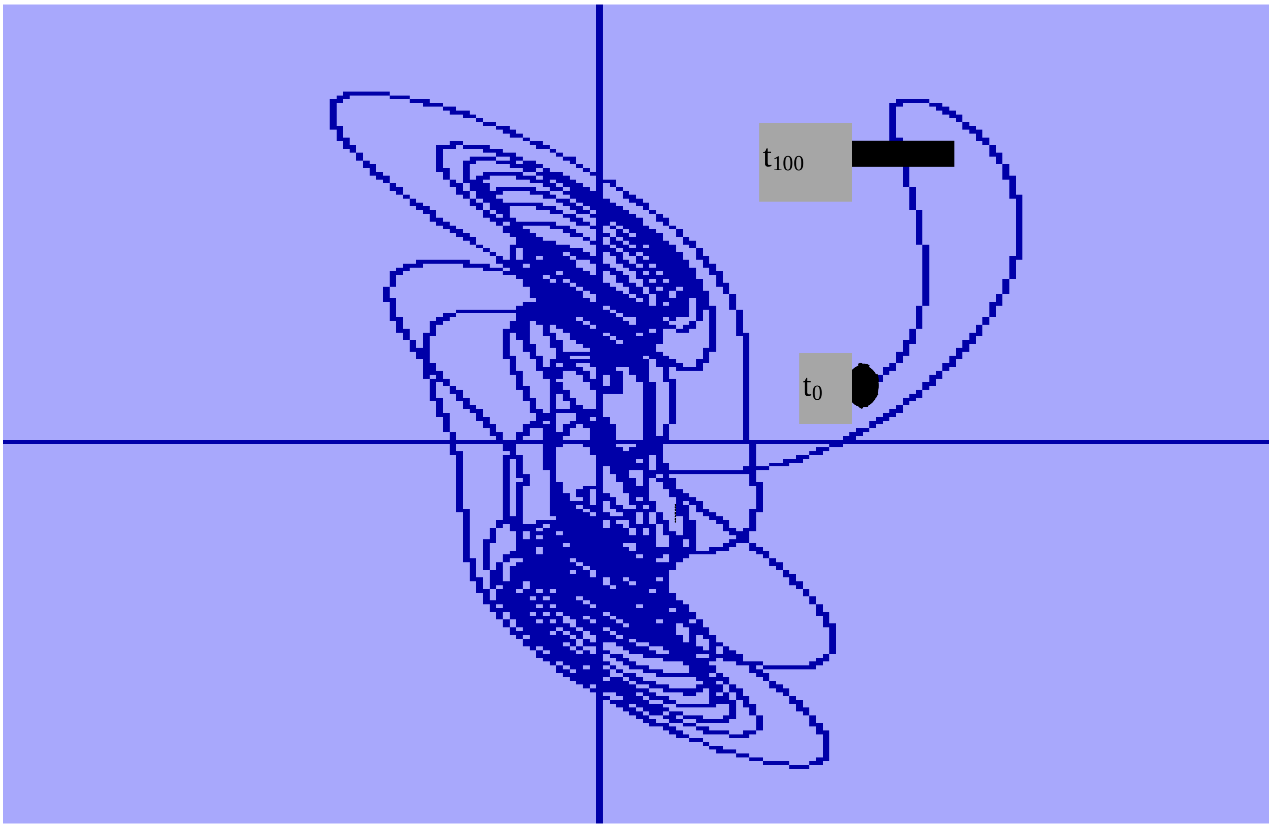
Trigonometric Version

The two-dimensional formal basis of Keynes Space [Q_t, M_t; t] for the three-dimensional Economic System [Q_t, M_t, P_t; t], as shown here, is the Cremona Equations.[11]

***Figure 2A: The Economy as an Strange Attractor Set in Keynes Space** [Medio 1992, Software **DMC**].*

[10] This is the alternative (and more generalized) version of Samuelson's Conservation Law [Sato and Ramachandran 1990 p. 57].

[11] According to Rosane's Theorem, these equations possess the conservative property in two-dimensional space [Tamari 1995].

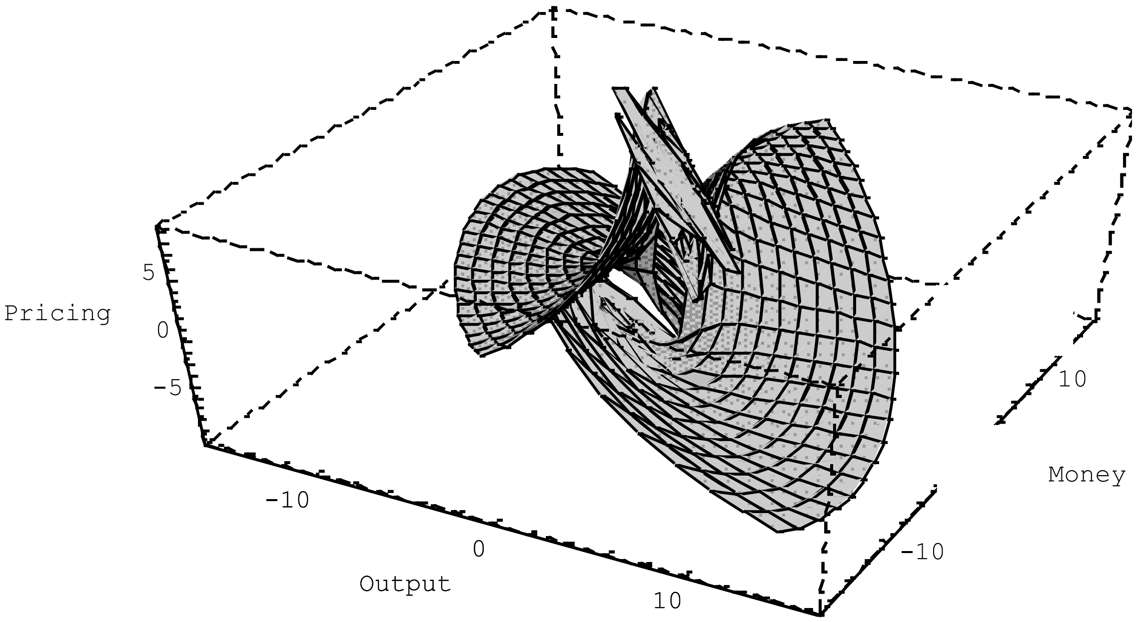


$$Q_{t+1} = (Q_t - a\,M_t)\cos(P_t) + b\,M_t \sin(P_t) \quad \textbf{\textit{Cremona}}$$

$$M_{t+1} = (Q_t + c\,M_t)\sin(P_t) + d\,M_t \cos(P_t) \quad \textbf{\textit{Equations}}$$

$$P_{t+1} = E + G\,P_t + \psi[(1-U)M_t \;/\; (1-I)Q_t] \quad \textbf{\textit{Pricing}}$$

Figure 2B: ***The Manifold of the Equations set in p. 17, produced by Mathematica®.***




```
ParametricPlot3D[
{(Q - 1.013 M) Cos[M/Q] - 0.011M Sin[M/Q],
 (Q + 0.02 M) Sin[M/Q] + 0.996 M Cos[M/Q],
 6ArcTan[((1 - 0.05) M) / ((1 - 0.05)Q)]},
{Q, -12,12}, {M, -11, 11},
PlotPoints -> {25,25},
AspectRatio -> GoldenRatio^(-1),
Background -> GrayLevel[1], LightSources ->
{{{-1,-1, 1}, GrayLevel[0.4]},
{{  1, 1, 1}, GrayLevel[0.4]},
{{ 0, 1, 1}, GrayLevel[0.4]}},
BoxStyle -> Dashing[{0.01, 0.01}],
AxesLabel -> {"Output", "Money", "Pricing"}];
```

The third equation (the Pricing) of the three-dimensional representation is the feedback equation.

Here **a, b, c,** and **d** denote the sensitivities/elasticities of money and the output with respect to each other; **E** the exchange rate, **G** the degree of indexation, **Ψ** trigonometric function (such as arctan(•)), **U** the unemployment rate, and **I** the interest rate.[12]

As a matter of fact, we now have three oscillators: output (Q_t), money (M_t) and pricing (P_t) connected together in a set of equations.

Each country has its characteristic coefficients **a, b, c, d, E, G, U,** and **I.** The coefficients **a, b, c,** and **d** are derived econometrically from the difference equations (p. 14). The coefficients **E, U**, and **I** are given in the national statistic, and **G** is a guess.

Figure 2 was computed by introducing the trigonometric version (p. 17) in **DMC** and **Mathematica®** software [Medio 1992, Wolfram 1996].

The starting point (t_0) is at: Q=1, M = 0.33 , and P=0.33. The trajectory computed 11,000 iterations (years), but after 100 iterations the trajectory was swept into a strange attractor, and became irrelevant for my study. Therefore, I cut off the trajectory at point $t_{n=100}$. The coefficients **a**=1.013, **b** = - 0.011, **c** = 0.02, and **d**=0.996 were taken as the averages of Table 1 (p. 29), **E** = 0, **G** = 0.01, **Ψ** = arctan(•), **U** = 0.05, and I = 0.05.[13]

Figure 2 (p. 17) is actually the way the Global Economy behaves, and each country is a variant of this figure (see Israel and Mexico - Case Study, p. 45). Most of the time we are interested only in the first (or sometimes also in the second) iteration. The strange attractor is like a *Sun* which governs our economy.

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[12] The unemployment and the interest rates (**U** and **I** respectively**)** determine the proportion of money and output, respectively, that are brought to the markets for exchange, I have coined them Gresham's Coefficients [Tamari 1991 p. 16].

[13] This dynamic system is very rich, and the simulations do not reach or exhaust its power and richness. I used relatively simple simulations to present the main economic point - the trajectory between the points t_0 and t_n in Figure 2A, p. 17 and its manifold in Figure 2B, p. 18.

Symmetrical Economic System

The economic system is quadratic in the quantities of output and money [Q_t, M_t], and linear in the rates of growth and printing [q, m]. If the system is not quadratic in the variables and linear in the rates, it will have nonlinear dynamics in the short run only. In the long run the system will evaporate (if the exponent is greater than 2) or vanish and become singular (if the exponent is less than 2).

The advantage of my approach, as represented here, is its ability to forecast the direction in which the economic system is moving, and to evaluate the economic situation of any country, at any given time, by use of the Jacobian **J**. The Jacobian coefficient serves as a kind of a '*thermometer*' for the economic 'health/sickness' of a country.

Additionally, my approach has the capacity to overcome three fundamental theoretical problems, with which the conventional approach cannot cope: *aggregation***,** *allocation* and *distribution*. A disadvantage, however, is that my approach requires explanation of the individual behavior on a macro-foundational basis instead of vice versa. The individual is analogous to a cobblestone in a road.

Aggregation

My approach allows for representation of the conservative economy in a complex presentation. Thus, aggregation (or disaggregation), can be carried out for any number of individuals constituting the economic system in question, varying from one individual, to families, firms and sectors of the economy, without the need for a representative individual or (cardinal or ordinal) utility or profit functions.

Allocation and Distribution

The problems of allocating production factors in the production process, on the one hand, and distributing output among the individuals participating in its production, on the other, are solved simultaneously, in the same space.

Stabilization

An economy in which real money balances begin to decline, after having built up in time to twice the normal figures, the velocity of circulation and the nominal value of money have risen, and the real value of the currency has fallen, must prepare itself for economic reform. The task of the reform is to steer the economy back to an optimal position in Keynes Space. An optimal position in Keynes Space means a favorable ratio of quantity of money (M1) to output (Q, measured as G.D.P. at constant prices), in the range $1/3 \leq M1/Q < 2/3$.[14]

The longer a reform is postponed, the more drastic the curbing measures necessary to remedy the ensuing situation. The required reforms must include five measures:

1. Balancing the budget.
2. Monetary restraint.
3. Settlement of debts.
4. Currency normalization, erasure of zeros.
5. Adjustment of the exchange rate, (relative to some foreign currency).

Such measures are indeed common, in one form or another, in many stabilization programs. The success of any economic reform depends first and foremost, on the ability to control the money-printing mechanism and to restore it to its proper dimensions (see Table 4, p. 38).

Inflationary potential emerges in the early stage of excess printing, during which time the issuer (the government of a country) may feel that it is getting "something for nothing". Ultimately, the trade deficit assumes crisis proportions, the national budget collapses, unemployment increases, and the public feels that its assets are dwindling to zero and that it is being short-changed.

Excess printing distorts the market's production and exchange mechanisms. Proper monetary adjustment, relative to growth, is the key to achieving the optimal economy and a prosperous one. What emerges is a multivariable feedback system - an economic domino game:[15]

A continuous budget deficit, financed by the borrowing and printing of money, weakens the monetary balance and forces up pricing (price level); this in turn depreciates the value of money and causes the trade deficit to increase. The rise in pricing and the resulting increase

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[14] The origin of this range is based in empirical and theoretical reasoning. The author believes that underlying this finding is the *golden rule.*
[15] Underlying the domino game is a logistic mechanism in which all the critical changes, from acceleration to collapse,

in exchange rate cause loss of taxes, increased government spending and a further rise in the budgetary and foreign deficit. At a certain stage, Gresham's Law[16] begins to operate and further aggravate the situation. The public loses faith in the domestic currency and the exchange rate soars.

In this multivariable feedback process, the economy experiences a slow transition, the rate of which depends on the level and scope of indexation, from stability to inflation and from inflation to hyperinflation. With the variables accelerating at an increasing rate upon each circulation of the feedback loop (Table, 4 p. 38).[17]

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[16] Gresham's Law: 'Bad money drives out good'. That is, bad money drives good money out of the markets, and good money drives bad money out of people's pockets.

[17] Some authorities believe that indexation neutralizes the harmful effects of inflation. In reality, however, it does not neutralize the damage due to inflation (tax), but only postpones it until a later time and transfers it from stronger to weaker elements of society. Strong elements use indexation to shield themselves, at the expense of others, against inflationary damage; while the weak are powerless to "index" (shield) themselves.

Empiric

In this section, I will present an empirical verification of the conservative law **J** = 1, and of the symmetry rule **p = m - q**. [18]

The Conservative Law

Table 1 (p. 28) lists the coefficients **a, b, c,** and **d** of the difference equations (p. 14) in 45 countries for the period from 1960 to 1996.[19] We can see on the last row of Table 1 (p. 29) and in Figure 3 (p. 32) that the Jacobian values of the countries are almost normally distributed around one (1). This distribution changes each year, but still we can see stability in the character of the distribution.

Table 1: ***The Jacobian Coefficients***

Country	a	b	c	d	J
Australia	1.033	- 0.015	- 0.015	1.018	0.997
Austria	1.019	- 0.022	0.025	0.978	1.052
Belgium	1.034	- 0.040	0.049	0.944	0.978
Canada	1.017	- 0.020	0.032	0.970	0.987
Colombia	1.007	- 0.004	0.019	1.008	1.015
Costa Rica	1.008	- 0.007	0.011	1.003	1.011
Cyprus	1.011	- 0.005	0.028	0.979	0.990
Denmark	1.011	- 0.011	0.027	0.981	0.992
Dominican Republic	1.006	- 0.003	- 0.006	1.033	1.038
Ecuador	1.008	- 0.005	0.015	1.025	1.033
El Salvador	1.002	- 0.002	- 0.002	1.019	1.021
Finland	1.043	- 0.021	0.057	0.978	1.021
France	1.020	- 0.022	0.043	0.956	0.976
Germany	1.010	- 0.008	0.014	0.995	1.005

[18] At this stage, I do not know of any statistical tests that are intended to examine dynamic systems whose dynamic regime changes. It is also impossible to implement a simulative examination for a constant parameter in economics [Lorenz 1993, Ch. 6]. The examination I have adapted is: are economic systems (countries), as they are expressed in I.F.S.Y empirical findings, found *most of the time* around the theoretical value, and do they have what is called the 'ergodic property' ?

[19] Countries were chosen from a sample as large as possible. For many countries there are not enough recent data (e.g. China and the former Soviet Union). For a number of countries, inflation is so high that data at a uniform year base cannot be reconstructed (e.g. Bolivia and Chile). In some countries the data seem insufficient because the country's population is too small, or data for too many years are missing.

Ghana	1.003	0.002	0.030	1.023	1.026
Greece	1.015	- 0.018	0.019	0.997	1.012
Guatemala	1.010	- 0.009	- 0.006	1.029	1.039
Honduras	1.010	- 0.009	0.004	1.027	1.038
India	1.001	0.007	0.006	1.014	1.015
Indonesia	1.003	0.004	0.067	0.934	0.936
Ireland	1.004	0.002	0.014	0.995	0.998
Israel	1.007	- 0.005	0.030	1.007	1.014
Italy	1.014	- 0.013	0.039	0.969	0.983
Japan	1.014	- 0.033	0.026	0.942	0.956
Korea	1.011	- 0.006	0.042	0.964	0.975
Mexico	1.007	- 0.007	0.009	1.022	1.029
Morocco	1.036	- 0.013	0.101	0.962	0.997
Myanmar	0.993	0.012	- 0.008	1.023	1.016
Nepal	0.997	0.009	0.014	0.999	0.996
Netherlands	1.019	- 0.020	0.028	0.978	0.997
New Zealand	1.008	- 0.007	- 0.007	1.023	1.031
Nigeria	1.011	- 0.011	0.001	1.022	1.033
Norway	1.010	- 0.007	0.016	1.003	1.014
Philippines	1.010	- 0.010	0.018	1.007	1.018
Portugal	1.011	- 0.010	0.011	1.006	1.018
Singapore	1.033	- 0.030	0.031	0.975	1.008
South Africa	1.012	- 0.013	- 0.006	1.023	1.036
Spain	1.018	- 0.019	0.031	0.975	0.993
Sri Lanka	1.008	- 0.006	- 0.001	1.014	1.022
Sweden	1.017	- 0.017	0.040	0.965	0.982
Switzerland	1.031	- 0.038	0.054	0.939	0.970
Turkey	1.006	- 0.004	0.003	1.046	1.052
United Kingdom	1.005	- 0.002	0.011	1.012	1.017
United States	1.014	- 0.014	0.010	0.996	1.010
Venezuela	1.010	- 0.011	- 0.003	1.060	1.070
Average	1.013	- 0.011	0.020	0.996	1.009

Source: Processed by the author from data of [I.F.S.Y. 1997].

My conjecture is that the economic system tends to maintain what has been coined by Ornstein and Weiss as 'Stochastic Stability' and by me, following their example, as 'Economic Stability' [Ornstein and Weiss 1985].

Please note the minus sign (-) of **b** in most of the countries in Table 1. This is compatible with the hyperbolic structure of the economy as represented by the function $W = (Q^2 - M^2)^{1/2}$, and is also required by the Cremona Equations, which need to preserve the economic space.

It is well known that dynamic systems where the Jacobian coefficient equals one (1) are *conservative systems*. They conserve *some* quantity.
When the Jacobian Coefficient of a country is greater than one (1**)** the situation is inflationary, and a restraining monetary policy must prevail. When the Jacobian Coefficient is less than one (1**)** the situation is deflationary, and an expanding monetary policy must prevail.

The Keynes Theory has been developed for the deflationary situation in which **J** < 1 (see Table 2), and indeed recommends expansion of the demands.

Table 2: ***The Jacobian Coefficients, USA, (1901 -1933)***

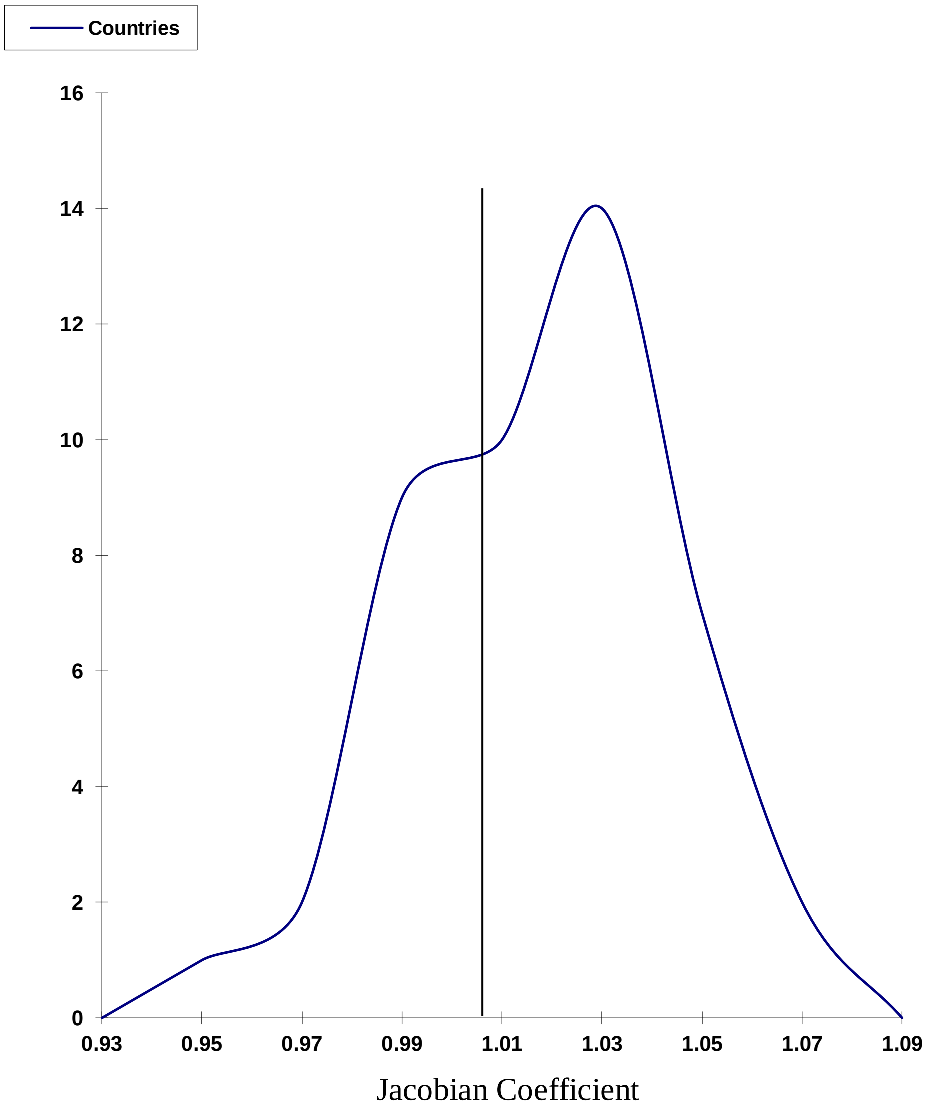
Years of Regression	**a**	**b**	**c**	**d**	**J**
1901 - 1924	1.012	- 0.011	0.031	0.962	0.974
1901 - 1925	1.012	- 0.012	0.029	0.966	0.978
1901 - 1926	1.010	- 0.008	0.031	0.962	0.972
1901 - 1927	1.012	- 0.010	0.033	0.956	0.967
1901 - 1928	1.012	- 0.012	0.035	0.952	0.964
1901 - 1929	1.011	- 0.010	0.036	0.949	0.961
1901 - 1930	1.016	- 0.021	0.038	0.943	0.959
1901 - 1931	1.019	- 0.028	0.041	0.936	0.955
1901 - 1932	1.023	- 0.040	0.046	0.925	0.948
1901 - 1933	1.024	- 0.041	0.048	0.921	0.944

Source: Processed by the author from data of different sources.

Please note in Table 2 the minus sign (-) of **b**, the coefficients at the year 1926 and the reduction of **J** after the year 1925.

Figure 3: **The Distribution of the Jacobian Coefficient Among the Countries,**
(45 countries, 1960 - 1996)

Jacobian	0.93	0.95	0.97	0.99	1.01	1.03	1.05	1.07	1.09
Countries	0	1	2	9	10	14	7	2	0



The Symmetry Law

A conservative system, which is also linear in its rates, is a symmetric system. Growth and printing (and, in parallel, investment and birth) are *translations* of the system, while inflation (and, in parallel, productivity) are its *rotations*. Taken together, therefore, they comprise the symmetrical group of the system.[20]

The empirical test to confirm (or refute) the linear relationship among growth, printing and inflation involves determining the deviation of the theoretical inflation $\mathbf{p}^*$ (= $\mathbf{m}$ - $\mathbf{q}$) from the measured inflation **p** of the various countries.

Table 3 (p. 34) lists average increases in output (gross domestic product at constant prices, GDP), money (money supply, M1), and pricing (consumer price index, CPI) in 45 countries for the period from 1960 to 1996.

Table 3: *Average Growth of Output (**q**), Money (**m**), Pricing (**p**), Theoretical Inflation (**p***) and Deviation (**d**), (*45 countries, *%*).

Countries	q	m	p	p^* = m-q	d = p^*-p
Australia	3.76	9.58	6.12	5.82	- 0.30
Austria	3.28	7.37	4.21	4.092	- 0.12
Belgium	2.91	5.35	4.53	2.44	- 2.08
Canada	3.76	9.47	4.96	5.72	0.76
Colombia	4.61	24.26	19.38	19.66	0.28
Costa Rica	4.56	17.99	14.14	13.43	- 0.71
Cyprus	5.44	11.07	4.55	5.63	1.08
Denmark	2.75	10.15	6.31	7.39	1.09
Dominican. Rep.	4.21	15.93	12.44	11.72	- 1.72
Ecuador	4.52	25.07	20.12	20.55	0.43
El Salvador	3.44	11.56	10.26	8.12	- 2.15
Finland	3.15	11.39	6.640	8.24	1.60
France	3.30	8.53	5.87	5.23	- 0.64
Germany	3.16	8.45	3.34	5.29	1.94
Ghana	2.60	28.91	28.90	26.32	- 2.58
Greece	3.99	16.66	11.77	12.67	0.90
Guatemala	4.00	13.08	9.06	9.08	0.02

[20] When one is dealing with different kinds of symmetric production functions, the parallel economic term is *holotheticity* [Sato and Ramachandran 1990, p. 37].

Honduras	3.99	12.46	8.39	9.47	1.08
India	4.44	12.85	8.05	8.41	0.35
Indonesia	5.94	37.81	32.28	31.87	- 1.41
Ireland	4.49	9.89	7.57	5.40	- 2.17
Israel	6.72	42.82	36.54	34.10	- 2.43
Italy	3.43	12.96	8.34	9.53	1.19
Japan	5.58	11.21	4.810	5.63	0.82
Korea	8.64	22.71	11.01	14.06	3.05
Mexico	4.34	29.85	24.54	25.51	0.97
Morocco	4.19	11.74	5.91	7.55	1.64
Myanmar	3.30	11.58	10.91	8.28	- 2.62
Nepal	3.42	15.78	9.53	12.36	2.83
Netherlands	3.31	8.225	4.31	4.91	0.60
N. Zealand	2.59	11.17	7.743	8.58	0.84
Nigeria	4.06	21.28	18.07	17.22	- 0.85
Norway	3.84	11.51	6.03	7.67	1.64
Philippines	4.02	14.67	11.13	10.65	- 0.48
Portugal	4.41	13.86	11.87	9.45	- 2.42
Singapore	8.55	10.35	3.25	1.80	- 1.45
South Africa	3.17	14.91	9.44	11.74	2.33
Spain	4.07	13.66	9.23	9.59	0.36
Sri Lanka	3.94	12.31	8.42	8.37	- 0.05
Sweden	2.48	8.89	6.35	6.41	0.06
Switzerland	2.27	5.75	3.71	3.48	- 0.23
Turkey	4.88	37.61	33.31	32.73	- 0.58
United Kingdom	2.30	11.27	7.20	8.97	1.76
United States	2.90	5.92	4.74	3.02	- 1.73
Venezuela	3.47	20.93	16.58	17.46	0.88
Average	4.0	15.3	11.22	11.30	0.08

Source: Processed by the author from data of [I.F.S.Y. 1997].

We can derive the econometric equation from Table 3:

$$\boldsymbol{p} \quad = \quad -\boldsymbol{0.25} + \boldsymbol{1.022\, m} - \boldsymbol{1.041\, q}$$

$$(\boldsymbol{t.s.}) \qquad\qquad (35) \qquad (-5.7)$$

$$\boldsymbol{R}^{*} = 0.968, \quad \boldsymbol{DW} = 2.42, \quad \boldsymbol{F} = 667$$

It is evident from the last row of Table 3 that the mean deviation between theoretical and actual inflation, for all countries in the sample, is only 0.08 point percent (0.74% deviation).

The distribution of this deviation is similar to the normal distribution but with a right-hand tail (see Figure 4). This indicates a tendency of the countries to generate excess money, which is not yet brought to the market and therefore has no effect on pricing [Bruno 1993, p. 124].

In the short run, this "monetary excess" causes marginal growth. This is the case as long as the system does not go beyond a certain parametric limit which has yet to be determined (see Table 4, p. 38); but when it does, collapse occurs.

Figure 4: *The Distribution of the Deviation of Theoretical Inflation (p^*) from its Measured Inflation (p) in point percent.*

(45 countries, 1960 - 1996)

Deviation	-2	-1	0	0.6	1.4	2.7	3.1
Countries	7	2	9	9	10	5	3

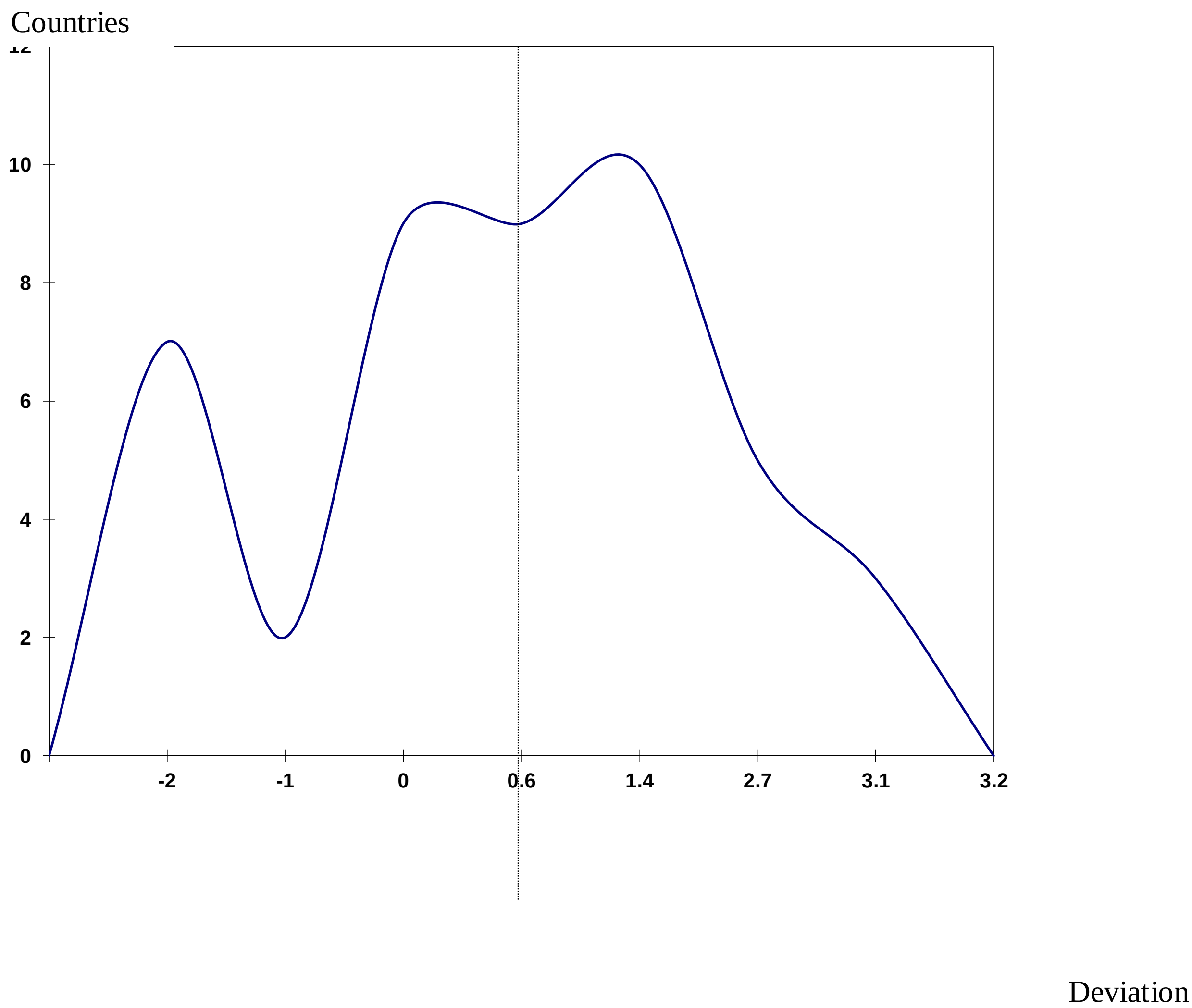


Table 4: Rates of Growth and Printing, And the Dynamic of the Economy (percentages, 1960-1996).

Variable	**Stable Area (Fixed Point)**	**Ascending (Periodic)**	**Acceleration (Quasi-periodic)**	**Demoralization (Chaos)**
Growth **q**	0 - 4	4 - 5	5 - 6	6 +
Printing **m**	0 - 10	11 – 15	16 - 40	41 +

At p* > p, inflation is suppressed, and at p* < p, it is released. The harmonic state is when **p = 0**, **m = q,** and **P =** $\mathbf{P^0}$. The two other states of disharmony are when m < q and we have the deflationary state; and when m > q and we have the inflationary state.

The countries wander around the optimum, as if in search of it. The economic process is thus like an evolutionary iterative process of trial-and-error, guided by pricing, aimed at finding an optimum.

Inflation and Stabilization

There is a saying:

Inflation occurs when too much
Money is chasing too few goods

In short, this summarizes my theory. Formally speaking;

$$p = m - q$$

Where **p** = inflation (the rate of change of pricing), **m** = printing (the rate of change in money), and **q =** growth (the rate of change in output).

An autonomous economy is at its optimum when **p = 0;** that is when *pricing* is stable (though relative prices may vary), inflation is zero, and printing equals growth [Tamari 1995]. This is the harmonic economy, each economic unit: producer, agent, consumer, family, firm, government and country is able to find its optimum.

Bruno points to this optimum (of the quantity of money) as relying on the quantitative equation (assuming a fixed velocity of circulation), and notes that the optimal quantity of money:

> "*Is best left to the discretion of the central bank, ..., rather than create[ing] a potential source of deficit financing. Otherwise, political pressures on the central bank to finance the deficit may prove destructive*" [Bruno 1993, p. 139, n. 23].

In my opinion, the decision of the optimal quantity of money should no longer be left to anyone's discretion, but rather imposed on the system as an operating rule by legal means.

My proposal is to replace the Nominal Anchor, in which the exchange rate is exogenously fixed, in order to "anchor" the system to a stable point (just as a ship is moored in port or a horse tethered to a pole), by a "Bedouin Knot." setting the rate of printing equal to the growth rate (**m = q**).[21]

The disadvantage of the Nominal Anchor is that it dictates tying the system to a certain exogenous parameter of the system, without knowing, theoretically or empirically, whether that parameter is optimal or even relevant for the system.

A Bedouin Knot, secured immediately after the first, curbing stage of the reform, will give the system (and its economic units) the time necessary for a better adjustment of economic variables than when the reforms are based on a Nominal Anchor, which does not allow for flexibility.

I am not claiming that stepwise reform is superior to instantaneous reform [Bruno 1993, p. 126]. I, too, believe that it is preferable at the first stage to curb inflation, impose a rough fit on the nominal variables, and only later to replace this by a Bedouin Knot.

As a first, curbing stage, a rough fit is carried out, anchoring the system to an exogenous nominal variable - as if it were activating brakes. Immediately thereafter, once the real balances have been adjusted, comes the second stage, in which the Nominal Anchor is abandoned and the system is adjusted through a Bedouin Knot [Patinkin 1993, p. 117].

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[21] The Bedouin (among others) tie the two front legs of their animals (camels, horses or asses) together and send them out to pasture. This permits maximal feeding of the animal at a minimal distance.

Conclusion

An autonomous economic system, i.e., a country, tends to be a *conservative* and *symmetrical system* in Keynes Space [Q_t , M_t ; t], and can therefore be represented as a complex numbers system. This presentation makes it possible to *aggregate* (or disaggregate) the system at all levels, from the individual to the most general aggregate (and vice versa). It also offers a simultaneous solution of the problem of *allocation* and *distribution* of useful sources in the market.

Inflation is a symmetrical phenomenon; both money and output, which are brought to the market and replace one another, have a part in its formation. Because of the relativity of the measure (money) and the entity to be measured (output), we tend to view inflation as a monetary phenomenon.

Replacement of the Nominal Anchor (fixing an exogenous exchange rate ($\mathbf{P_{ex} = P_{ex}^0}$)) by a Bedouin Knot (setting the rate of printing equal to the growth rate ($\mathbf{m = q}$)) after initial curbing steps is consistent with both theory and empirics as represented here. This old-new rule enhances the recovery of the economy and moderates the adjustment pressures.

Appendices

Israel and Mexico - Case Study[22]

This section responds to the question "Is Israel's economy subject to the same risk situations as Mexico's economy?" This question, which Liviatan examined comparatively [Liviatan 1995] is examined here using the methodology developed by the author [Tamari 1991, 1995].

This method's central motif, in contrast to the regular method, is its regard for money as both a production and measuring factor; as a production factor, money is therefore not neutral.

The method assumes that one analytical system (Basic Equations) exists in all economies and only equation coefficients differ from country to country.

The advantages of this approach are its globalness, the relative ease with which empirical findings are attained, its ability to make international comparisons, and its good forecasts. The disadvantages are the depth and mathematical knowledge necessary to understand and operate this approach and the fact that it is still in a very early developmental stage.

The central finding: The Israeli economy (and the Mexican one) is still in a high risk area and therefore, it is necessary to adopt very cautious budgetary and monetary policies.

Israel, like Mexico, is still not immune to shocks; each shock may easily turn into crisis, instead of being absorbed as a temporary slowing of growth. Moreover, it is conceivable that the 1985 stabilization program did not succeed as believed.

This conclusion is consistent with that of Liviatan:

> *"The great similarity between the process that led to the crisis in Mexico and the development lines of the Israeli economy at this stage hint that today, there is a reason to take preventative measures in the form of more flexibility in the exchange rate and restraint in the budget. The flexibility will decrease the potential for a run*

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[22] The author believes that the correct geopolitical comparison for Israel is to post-World War I Germany. The similarity is in the struggle for security, inflation, and the stabilization that followed. Since an article on this subject has been written (Tamari 1985), we know who 'is playing' Schact (Bruno) and to whom the role of Rathenau was assigned (Rabin, Z"L), … .

on the reserves and the budgetary restraint may contribute to a real devaluation that will also reduce the possibility of a future crisis" [Liviatan, p. 27].

The following table (Table 5) and diagram (Figure 5) present the latest findings produced from the latest available international statistics [I.F.S.Y. 1997].

The diagram shows that Israel's trajectory (the solution path) is to the left of the lead of one (**1**) on the output axis. This teaches that Israel's current, dynamic situation (for data until 1996) is better than Mexico's, but the two countries are still in a risk area, an area where each shock accelerates instead of being absorbed. This is the case even though both countries' situations have improved greatly since 1989 [Tamari 1991, p. 62].

An additional, interesting finding in the diagram is that in the future, the economy of the USA is expected to face a similar change to that which has been forecast for the Japanese economy from a previous examination [Tamari 1990, p. 232]. That is, the economy of the USA, if it continues on its current path, will face a significant slowdown.

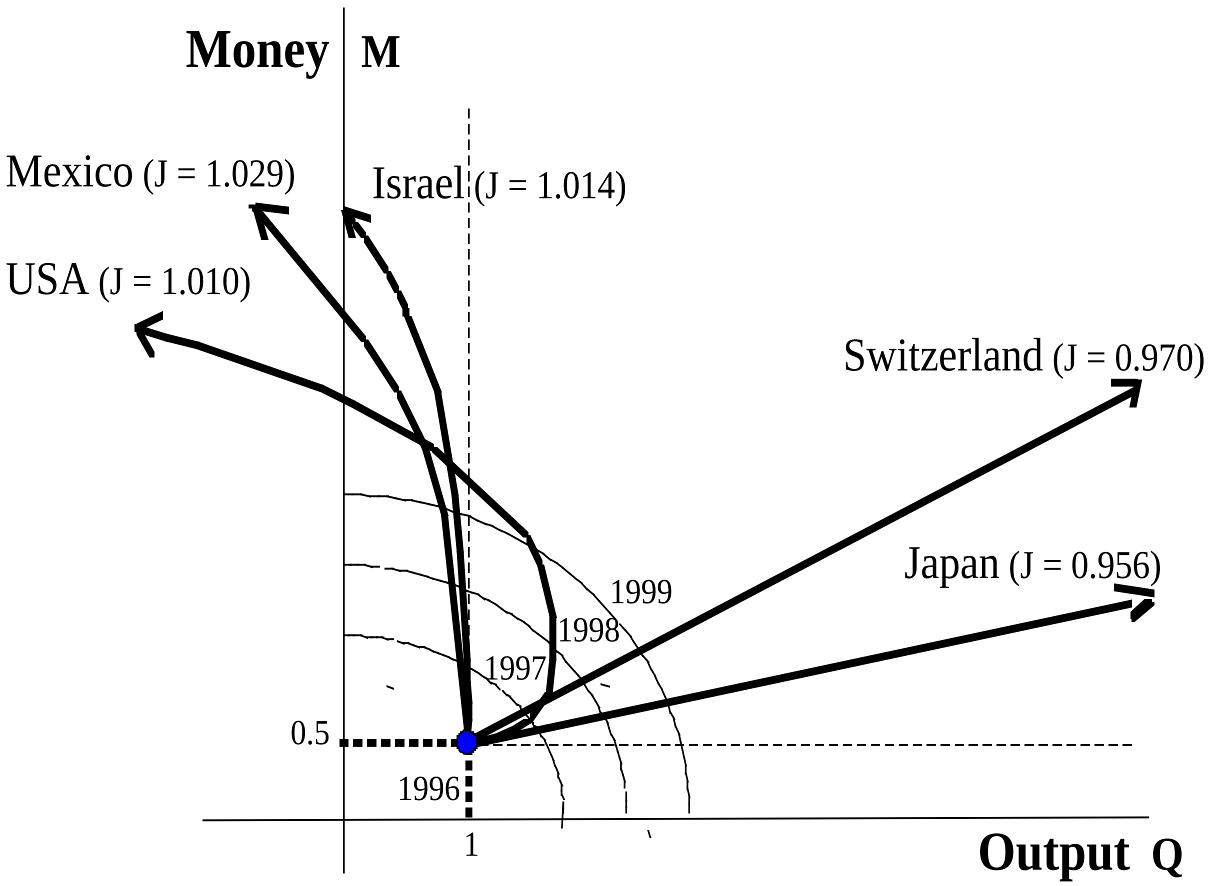
Table 5: *Symmetry and Conservation: Israel, Japan, Mexico, Switzerland and the US. (1960 - 1996)*

	Israel	**Japan**	**Mexico**	**Switz.**	**USA**
Symmetry Law: Rate of change (multi-year averages, %)					
Growth , **q**	6.7	5.6	4.3	2.3	2.9
Printing, **m**	42.8	11.2	29.9	5.8	5.9
Inflation, **p**	36.5	4.8	24.5	3.7	4.7
$p^* = m - q$	34.1	5.6	25.5	3.5	3.0
$d = p^* - p$	- 2.4	0.8	1.0	- 0.2	- 1.7
Conservation Law: Coefficients					
a	1.007	1.014	1.007	1.031	1.014
B	- 0.005	- 0.033	- 0.007	- 0.038	- 0.014
C	0.030	0.026	0.009	0.054	0.010

D	1.007	0.942	1.022	0.939	0.996
J = ad - bc	1.014	0.956	1.029	0.970	1.010

Figure 5: *The Dynamic Courses of the Economies of Israel, Japan, Mexico, Switzerland and the USA.*

(1996 and Onwards)



The number in parentheses is the Jacobian value **J.**

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